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# A Prelude to Ultrahigh-Energy String Excitation

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## Summary

The configuration of highly-excited string states is recapitulated through ultrahigh energy, perturbative string scattering without direct recourse to the thermal string amplitude in proper reference to the possible violation of unitarity bounds, the possible macroscopic nonlocality and the possible association with the newfashioned percolation scenario. The crucial role of the order parameter of the string/black hole correspondence is then sketched from the viewpoint of string cosmology.

In a precedent compendium of ours [1], ultrahigh energy aspects of perturbative string scattering were surveyed on the basis of the thermal Virasoro formula [2] as well as the thermal Veneziano formula [3] in proper respect of the possible violation of unitarity bounds such as the Froissart bound [4] and the Cerulus-Martin bound [5]. The configuration of highly-excited string states was then sketched after ref. [2] and ref. [6] from the standpoint of string cosmology [7-12] with regard to the possible macroscopic nonlocality due to string extendedness and/or strong gravitational effects. The possible association with the newfashioned percolation scenario of the black hole ensemble was also touched upon. In the present communication, the previous observation [1] on violation of unitarity bounds, macroscopic nonlocality and association with the newfashioned percolation scenario is affirmatively epitomized through ultrahigh energy, perturbative string scattering without direct regard to the thermal Virasoro amplitude as well as the thermal Veneziano amplitude. The crucial role of the order parameter of the string  $\rightleftharpoons$  black hole transition is then illustrated from the viewpoint of string cosmology.

The non-planar, four-tachyon tree amplitude  $V_{cl}(s, t, u)$  of closed bosonic strings is described as [13, 14]

$$V_{cl}(s, t, u) = g^2 \frac{\Gamma(-\frac{1}{2}\alpha_{cl}(s))\Gamma(-\frac{1}{2}\alpha_{cl}(t))\Gamma(-\frac{1}{2}\alpha_{cl}(u))}{\Gamma(-\frac{1}{2}[\alpha_{cl}(s) + \alpha_{cl}(t)])\Gamma(-\frac{1}{2}[\alpha_{cl}(t) + \alpha_{cl}(u)])\Gamma(-\frac{1}{2}[\alpha_{cl}(u) + \alpha_{cl}(s)])}, \quad (1)$$

where

$$\alpha(\zeta) \equiv \alpha_{cl}(\zeta) = 2\alpha_{op}(\frac{\zeta}{4}) = 2\alpha_{op} + \frac{1}{2}\alpha'_{op} \cdot \zeta = \alpha_{cl} + \alpha'_{cl} \cdot \zeta = 2 + \frac{\zeta}{4}, \quad (2)$$

$\Gamma$  reads the gamma function, and  $g$  is the coupling constant of the closed bosonic string. In addition, the tachyon trajectory function  $\alpha_{cl}(\zeta)$  [ $\alpha_{op}(\zeta)$ ] of closed [open] bosonic string satisfies the constraint

$$\alpha_{cl}(s) + \alpha_{cl}(t) + \alpha_{cl}(u) = -2 \quad [\alpha_{op}(s) + \alpha_{op}(t) + \alpha_{op}(u) = -1]. \quad (3)$$

The planar, four-tachyon tree amplitude  $V_{op}(s, t, u)$  of open bosonic strings is written in the form [15]

$$V_{op}(s, t, u) = \bar{g}^2 \{B(-\alpha_{op}(s), -\alpha_{op}(t)) + B(-\alpha_{op}(t), -\alpha_{op}(u)) + B(-\alpha_{op}(u), -\alpha_{op}(s))\}, \quad (4)$$

where  $B$  reads the Euler beta function, and  $\bar{g}$  is the coupling constant of the open bosonic string. Here, it is noted that  $\bar{g}^2 \sim g$  as a simple and natural consequence of the topological sewing machinery.

The high energy, fixed-angle behaviour of the Virasoro amplitude (1) is reduced to [16]

$$V_{cl}(s, t, u) \propto g^2(stu)^{-3} \exp \left[ -\frac{1}{4}(s \ln s + t \ln t + u \ln u) \right]. \quad (5)$$

Similarly, the high energy, fixed-angle behaviour of the Veneziano amplitude (4) turns into [15, 17]

$$V_{op}(s, t, u) \propto \bar{g}^2(stu)^{-3/2} \exp \left[ -\frac{1}{2}(s \ln s + t \ln t + u \ln u) \right]. \quad (6)$$

Asymptotic expressions (5) and (6) violate the Cerulus-Martin lower bound on the high energy, fixed-angle amplitude [5, 18]:

$$|F(s, \cos \theta)| \geq \exp[-f(\theta)\sqrt{s} \ln s] \quad (7)$$

with some appropriate function  $f(\theta)$ . The derivation of the lower bound (7) is inapplicable in the presence of infinitely-rising Regge trajectories such as eq. (2), however. The high energy, fixed-momentum-transfer behaviour of the Virasoro formula (1) is written in the standard Regge formalism as

$$V_{cl}(s, t) \sim \pi g^2 \frac{e^{-2-t/4}}{[\Gamma(2+t/8)]^2} \left(\frac{s}{8}\right)^{2+t/4} \left\{ i - \cot \left( \frac{\pi t}{8} \right) \right\}. \quad (8)$$

Thus the Virasoro amplitude (1) yields the total cross section of the form

$$\sigma_{cl}^T(s) \simeq \frac{1}{s} \text{Im} V_{cl}(s, t \simeq 0, u) \sim \pi g^2 l_s^4 s; \quad s \rightarrow \infty \quad (9)$$

up to a numerical factor in the tree approximation, where the fundamental string length  $l_s \sim \sqrt{\alpha'}$  is in association with the string tension  $(2\pi\alpha')^{-1}$ . The asymptotic expression (8) violates the Froissart upper bound on the high energy, forward amplitude [4, 18]:

$$F(s, t \simeq 0) \lesssim C s (\log s)^2 \quad (10)$$

with some appropriate constant  $C$ . The derivation of the upper bound (10) is inapplicable for closed string theory in association with massless modes in the sense of the absence of a gap, however. The cross section  $\sigma_{cl}^T(s)$  saturates the Froissart bound

$$\sigma_{cl}^T(s) \sim \pi l_s^2 \quad (11)$$

up to a logarithmic factor at around  $\omega_s = \sqrt{s} \sim (gl_s)^{-1}$ . The increasing cross section (9) is heuristically considered as arising from production of highly-excited, stretched strings of length  $\omega_s l_s^2$  at high energies. As already argued by Emparan *et al.* [19, 20], consequently, the production cross section of a long, highly-excited or equivalently highly massive, closed string state at mass level  $\omega_s$  is asymptotically described as eq. (9) in association with the dual symmetric, long-distance exchange of a short, light or equivalently massless, closed string state, corresponding to the single graviton exchange over a large distance. Let us now postulate validity of the Froissart bound (10) in string theory without loss of generality. It will then be possible to claim *à la* ref. [19] at least at sufficiently small coupling  $g$  that the production cross section (9) grows with  $s$  for  $l_s^{-1} \ll \omega_s < (gl_s)^{-1} \sim g\omega_c \sim M_{\text{P}} = l_{\text{P}}^{-1}$ , while remains constant at the saturated value (11) for  $g\omega_c \lesssim \omega_s \lesssim \omega_c \sim (g^2 l_s)^{-1}$ , where  $\omega_c$  reads the mass level at the string/black hole correspondence point,  $M_{\text{P}}$  and  $l_{\text{P}}$  are Planck mass and Planck length, respectively. Let us call to remembrance that  $\sigma_{cl}^T(s)$  can be identified at  $\omega_s \sim \omega_c$  with the production cross section of a black hole of the Schwarzschild radius  $l_s$ . The detailed discussion on the string/black hole correspondence as well as the total cross section  $\sigma_{cl}^T(s)$  for the case  $\omega_s \gtrsim \omega_c$  is referred to in the penultimate paragraph. Similarly, the Regge behaviour of the Veneziano formula (4) is expressed as

$$V_{op}(s, t) \sim \pi \bar{g}^2 \frac{e^{-1-t/2}}{\Gamma(2+t/2)} \left(\frac{s}{2}\right)^{1+t/2} \left\{ i + \tan\left(\frac{\pi t}{4}\right) \right\}. \quad (12)$$

Thus the Veneziano amplitude (4) brings forth the total cross section of the form

$$\sigma_{op}^T(s) \simeq \frac{1}{s} \text{Im} V_{op}(s, t \simeq 0, u) \sim \pi \bar{g}^2 l_s^2; \quad s \rightarrow \infty \quad (13)$$

up to a numerical factor in the tree approximation. The asymptotic expression (12) saturates the Froissart bound (10) up to a logarithmic factor. The production cross section of a long, highly-excited, open string state at mass level  $\omega_s$  is asymptotically described as eq. (13) in association with the exchange of a short, light, open string state instead of the single graviton exchange. Here, the open string cross section (13) is literally subdominant to the closed string cross section (9), *i.e.* the open string tachyon will be ineffectual as compared with the closed string tachyon, at sufficiently high energies such as  $\omega_s > (g^{1/2} l_s)^{-1} \sim g^{3/2} \omega_c$  in proper reference to the production mechanism of a highly massive, string state beyond mass scale  $g^{3/2} \omega_c$ . It may be stated parenthetically that the cross section  $\sigma_{op}^T(s)$  never attains the geometrical value

$\pi l_s^2$  at small string coupling  $g < 1$ , in sharp contrast to the cross section  $\sigma_{cl}^T(s)$ . The detailed behaviour of  $\sigma_{op}^T(s)/\sigma_{cl}^T(s)$  is left out of consideration in the present context, however.

As exemplified in ref. [1], the effective coupling constant squared  $g_{eff}^2$  of the closed bosonic thermal string is asymptotically reduced to

$$g_{eff}^2 \simeq \frac{g^2}{3} \left\{ \frac{e^{\beta\omega_s/2}}{e^{\beta\omega_s/2} - 1} \frac{e^{\beta\omega_t/2}}{e^{\beta\omega_t/2} - 1} + \frac{e^{\beta\omega_t/2}}{e^{\beta\omega_t/2} - 1} \frac{e^{\beta\omega_u/2}}{e^{\beta\omega_u/2} - 1} + \frac{e^{\beta\omega_u/2}}{e^{\beta\omega_u/2} - 1} \frac{e^{\beta\omega_s/2}}{e^{\beta\omega_s/2} - 1} \right\} \\ \sim g^2 \omega_s l_s \quad ; \quad s \rightarrow \infty \quad ; \quad t \sim 0, \tag{14}$$

in the standard dispersion theoretic approach based upon the thermofield dynamics [TFD] [21], where  $\beta = 1/kT$  and  $\omega_\zeta = \sqrt{|\zeta|}$ ;  $\zeta = s, t, u$ . Similarly, the effective coupling constant squared  $\bar{g}_{eff}^2$  of the open bosonic thermal string asymptotically turns into

$$\bar{g}_{eff}^2 \simeq \frac{\bar{g}^2}{3} \left\{ \frac{e^{\beta\omega_s}}{e^{\beta\omega_s} - 1} + \frac{e^{\beta\omega_t}}{e^{\beta\omega_t} - 1} + \frac{e^{\beta\omega_u}}{e^{\beta\omega_u} - 1} \right\} \\ \sim \bar{g}^2 \omega_s l_s \quad ; \quad s \rightarrow \infty \quad ; \quad t \sim 0. \tag{15}$$

It is of interest to mention that  $g_{eff}^2 \sim g^2$  [ $\bar{g}_{eff}^2 \sim \bar{g}^2$ ];  $\beta \rightarrow \infty$  and  $g_{eff}^2 \sim g^2(kT)^2$  [ $\bar{g}_{eff}^2 \sim \bar{g}^2 kT$ ];  $\beta \rightarrow 0$  at nonzero, finite values of  $s, t$  and  $u$ , reminiscent of the Atick-Witten formula [22] for the effective theory of closed [open] bosonic thermal strings. Consequently, the present argument of the configuration of highly-excited, closed string states will be in full agreement with the precedent thermodynamical investigation [1] of the thermal string ensemble based upon the TFD algorithm in the sense that  $g_{eff}^2 \sim g^2 \omega_s l_s \sim \omega_s/\omega_c$  plays the role of the order parameter of the string  $\rightleftharpoons$  black hole transition at asymptotically high energies in both approaches. In addition, it is noted that the criterion  $\bar{g}_{eff}^2 \sim \bar{g}^2 \omega_s l_s \sim g \omega_s l_s \sim \omega_s/M_P \sim 1$  will effectively describe the string coupling squared at the onset of the geometrical value (11), *i.e.* the saturated production cross section of a highly-excited, closed string state beyond the Planck mass scale as an immediate consequence of the topological sewing machinery *à la* non-planar fashion. It is now reminded, however, that  $\sigma_{cl}^T(s)$  and  $\sigma_{op}^T(s)$  attain the geometrical value  $\pi l_s^2$  at around  $\omega_s \sim g^{1/3} M_P$  and  $\omega_s \sim M_P$ , respectively, in the precedent thermodynamical paradigm [1] of the thermal string ensemble.

The configuration of highly-excited, closed bosonic strings is sketched after ref. [1] at trans-Planckian energies, *i.e.*

$$\omega_s > M_P = 1/l_P = 1/\sqrt{G} \sim 1/g l_s > 1/l_s \sim 1/\sqrt{\alpha'}, \tag{16}$$

from the viewpoint of string cosmology [7-12] based upon the hypothesized holographic principle and the conjectured correspondence principle. Here, the sufficiently small string coupling  $g \sim l_P/l_s < 1$  has been postulated in perturbative string scattering and use is made of  $\alpha'_{cl} = 1/2 \cdot \alpha'_{op} \equiv \alpha' \neq 1/4$  and  $G \neq 1$  besides  $c = \hbar = k = 1$  in the present context. Strong gravitational dynamics may lead to the possible failure of local quantum field theory on scales much larger than the Planck length  $l_P$  at ultrahigh energies when a given energy  $\omega_s$  is concentrated inside a closed trapped domain, *i.e.* a black hole, of the Schwarzschild radius  $R_S \sim \omega_s g^2 l_s^2 \sim (\omega_s/\omega_c) l_s$ . On the other hand, a highly-excited string of energy  $\omega_s$  can stretch over a distance  $\omega_s l_s^2$  and might yield macroscopic nonlocality much larger than the string scale  $l_s$  at ultrahigh energies. Such stringy nonlocality could prevent formation of black holes in high energy collisions, because the string energy distribution spreads out on scales  $\omega_s l_s^2$  large as compared to the supposed horizon of the Schwarzschild radius  $R_S \sim \omega_s g^2 l_s^2$ . There is no manifest indication for such long-string effects, however [23, 24]. The scattering amplitude is really dominated by the long-range gravity beyond a scale of tidal string excitation:  $l_D \sim \omega_s g l_s^2$  [23-25]. Significant tidal excitation might cause some sort of stringy nonlocality. Moreover, the tidal excitation scale  $l_D$  is larger than the supposed Schwarzschild radius  $R_S$ . There exists no manifest indication for such stringy nonlocality due to tidal string excitation, however [23, 24]. Accordingly, extendedness of the string will not cause long-distance nonlocal effects which interfere with formation of a closed trapped surface at ultrahigh energies  $\omega_s > \omega_c$ , at which the Schwarzschild radius  $R_S$  exceeds the string length  $l_s$ . Principal conclusions on unitarity and nonlocality are then epitomized *à la* [23, 24] as follows: Firstly, there will be no indication of macroscopic nonlocality intrinsic to extendedness of the string. Secondly, black hole formation will be inherent in strong gravitational dynamics without interference due to stringy nonlocality. Thirdly, the possible breakdown of locality at scales  $R_S$  will be inevitably associated with breakdown of gravitational perturbation theory at the black hole threshold. Finally, violation of asymptotic bounds for local field theory, such as the Froissart bound, may be intimately connected with macroscopic nonlocality intrinsic to gravitational nonperturbative dynamics.

As has often been emphasized by ourselves [26, 27], there appears the maximum temperature  $T_c \sim l_s^{-1}$  of string excitation, beyond which the thermal string amplitude is infrared divergent. The maximum temperature  $T_c$  is of the same order as the Hagedorn temperature  $\hat{T}_H \sim l_s^{-1}$  of the thermal string ensemble, beyond which the canonical partition function diverges for



sufficiently large values of mass. The string/black hole correspondence is then recapitulated as follows: The string entropy  $S_s \sim \omega_s l_s$  at mass level  $\omega_s$  turns out to be the same order as the Bekenstein-Hawking entropy  $S_{\text{BH}} \sim \omega_s^2 g^2 l_s^2 \sim (\omega_s/\omega_c) S_s$  of the corresponding black hole when the string length  $l_s$  becomes of the order of the Schwarzschild radius  $R_S \sim \omega_s g^2 l_s^2 \sim (\omega_s/\omega_c) l_s$ . If  $g^2 < (\omega_s l_s)^{-1}$ , *i.e.*  $\omega_s < \omega_c$ , then, the Bekenstein-Hawking entropy  $S_{\text{BH}}$  is smaller than the string entropy  $S_s$ , *i.e.* the Hawking temperature  $T_{\text{H}} \sim (\partial S_{\text{BH}}/\partial \omega_s)^{-1} \sim R_S^{-1} \sim (\omega_c/\omega_s) l_s^{-1}$  is higher than the Hagedorn temperature  $\hat{T}_{\text{H}} \sim l_s^{-1}$  and the string will spread out on scales much larger than the supposed horizon so that the black hole is depicted as a continuum string state. If  $g^2 > (\omega_s l_s)^{-1}$ , *i.e.*  $\omega_s > \omega_c$ , on the other hand, the Bekenstein-Hawking entropy  $S_{\text{BH}}$  is larger than the string entropy  $S_s$ , *i.e.* the Hawking temperature  $T_{\text{H}}$  is lower than the Hagedorn temperature  $\hat{T}_{\text{H}}$ , and the horizon will be much bigger than the string length scale so that the string energy is concentrated inside a closed trapped domain and consequently the string behaves as a black hole. Accordingly, the criterion  $g_{\text{eff}}^2 \sim g^2 \omega_s l_s \sim \omega_s/\omega_c \sim 1$  will effectively describe the string coupling squared at the string  $\rightleftharpoons$  black hole transition point. Thus  $g_{\text{eff}}^2 \sim g^2 \omega_s l_s \sim \omega_s/\omega_c$  plays the role of the order parameter of the string/black hole correspondence at asymptotically high energies. As a consequence, the critical temperature  $T_c \sim l_s^{-1}$  is naturally interpreted as the phase transition temperature at which the thermal string configuration turns into a localized black hole and *vice versa*. The production cross section  $\sigma_{\text{BH}}^T$  of a black hole at mass level  $\omega_s$  is geometrically written in the form

$$\sigma_{\text{BH}}^T \sim \pi R_S^2 \sim \pi \omega_s^2 g^4 l_s^4 \sim \pi (\omega_s/\omega_c)^2 l_s^2; \quad \omega_s \gtrsim \omega_c \quad (17)$$

which turns out to be

$$\sigma_{\text{BH}}^T \sim \pi l_s^2 \quad (18)$$

at the string  $\rightleftharpoons$  black hole transition point:  $g_{\text{eff}}^2 \sim g^2 \omega_s l_s \sim \omega_s/\omega_c \sim 1$ . It is of interest to note that the production cross section (18) is equal in magnitude to eq. (11), *i.e.* the saturated production cross section of a highly-excited string state at mass level  $\omega_s$  for  $g\omega_c \lesssim \omega_s \lesssim \omega_c$ , or equivalently  $(\omega_s l_s)^{-2} \lesssim g^2 \lesssim (\omega_s l_s)^{-1}$ . As a salient feature of our argument in ref. [6], the most probable microcanonical distribution of primordial black holes is self-consistently described at ultrahigh energies in asymptotically flat space through the so-called single-massive-mode dominance scenario in the sense that most of the mass, most of the charge and most of the angular momentum of the whole system converge on a single energetic black hole. In literal

agreement with the previous observation [1] on the string/black hole correspondence, therefore, it will be possible to conclude that the critical temperature  $T_c$  is intrinsically reminiscent of the newfashioned percolation temperature at which the multi-black hole ensemble coalesces into a single primordial black hole of the critical mass  $\omega_c$  and eventually transmutes into a single primordial string mode of the same mass. It still remains to be clarified in a nonperturbative fashion, however, whether or not the so-called percolation scenario *à la* Susskind *et al.* [12] is fully effectual in elaborating the possible linkage between self-gravitating single string states and multi-string states.

From the standpoint of string cosmology based upon the holographic principle and the correspondence principle, the present discussion of the configuration of highly-excited, string states is recognized at least at trans-Planckian energies as to be asymptotically equivalent to the precedent thermodynamical investigation [1] of the thermal string ensemble based upon the TFD algorithm in the sense that  $g_{eff}^2 \sim g^2 \omega_s l_s \sim \omega_s / \omega_c$  plays the role of the order parameter of the string/black hole correspondence at ultrahigh energies in both approaches with and without the direct aid of the thermal string amplitude. Consequently, it will be possible to claim that we have succeeded in lending active credence to validity of the previous observation [1] on violation of unitarity bounds established for local field theory, macroscopic nonlocality intrinsic to strong gravitational effects and relationship to the newfashioned percolation scenario of the black hole ensemble through ultrahigh energy, perturbative string scattering without direct recourse to the thermal Virasoro formula as well as the thermal Veneziano formula, where the substantial use has been made of the asymptotic equivalence  $g_{eff}^2 \sim g^2 \omega_s l_s \sim \omega_s / \omega_c$  of the order parameter of the string  $\rightleftharpoons$  black hole phase transition.

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