Symmetric Approach to Intertemporal Consumption Choice and Statewise Optimization under Uncertainty

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1: Introduction

One of the theoretical targets of uncertainty economics is to offer a framework according to which the value of the choice variable is selected by agent as deterministic. Thus the demand for securities as risky asset and the money stock as safe asset are both deterministically chosen by the agent in the portfolio choice theory. Similarly, the theoretical target of the intertemporal consumption choice model, when the market price variables (interest rate) and the budget (human capital + non-human capital) constraint are both uncertain, is to derive the current period consumption demand C_1 (and, presumably, the intertemporal consumption path as well) as a deterministic value, conditional on the currently available information.

The well-known standard approach to this problem (Hall[1978], Selden[1978, 79]) has employed PIH (permanent income hypothesis) as a common theoretical framework, and this paper follows the same line in this respect. The standard approach, however, has employed another common theoretical framework, namely that the current period consumption C_1 as an optimization target is non-stochastic but that the future consumption (C_2, C_3, *etc.*) is stochastic. The importance as well as restrictiveness of this "Asymmetry Postulate" (AP) does not seem to be fully recognized.

Selden, op.cit. is most explicit in adopting AP by assuming (a) that C_1 is certain and C_2 is uncertain so that (C_1, C_2) is "Certain-Uncertain Pairs", and that a preference ordering exists between these pairs[1]. Further, (b) "Risk Preference Independence[2]" is assumed so that the risk preference with respect to the uncertain C_2 is independent of the certain C_1. Concerning both (a) and (b), C_1 and C_2 are explicitly postulated as asymmetric variables. These assumptions concern C_1 and C_2 interpreted as optimizing

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[1] Kreps-Porteus[1978], also assuming "Certain-Uncertain Pairs", uses a common framework with Selden. Epstein-Zin[1989] apparently follows the same line. We note in passing that our approach belongs to what Kreps-Porteus, op.cit.,p.185-187 called "payoff approach", which is more restricted than Kreps-Porteus.

[2] Selden, op.cit., p.1053.

objectives: in other words, these assumptions logically precede the optimum choice behavior itself.

In contrast, Hall, op. cit. is not very explicit in adopting AP. However, we will show (see Section 5) that Hall's consumer is interpreted as maximizing the expected utility of consumption path not only under the budget constraint (stochastic) but also under an implicit constraint requiring C_1 be constant under any stochastic occurrence of the budget. Therefore, his model also adopts AP and virtually treats (C_1, C_2) as "Certain-Uncertain Pairs".

It is the consequence of AP that the standard approach is able to choose the optimum consumption path, the optimum C_1 in particular, as deterministic. The AP framework, however, is open to doubt, once we recognize that formally the same framework must be similarly applicable to any non-intertemporal consumption choice problem in which the price variables and the budget variable are uncertain. After all, the subscripts of C_1 and C_2 may well express not the time period but specific goods (say, apple and orange). It is questionable if one may plausibly adopt AP in this general case and assume that the demand for apple is non-stochastic but that for orange is stochastic. We must ask ourselves if there is another approach to solve the intertemporal consumption choice under uncertainty without adopting AP: this is the main purpose of this paper.

The intertemporal consumption choice under uncertainty is an optimization problem when the price and the budget variables, to be called "Source Uncertainty (SU)", are jointly distributed, generating many potential budget constraints. This article will assume that the consumer will optimize with respect to each and every stochastically possible occurrence (i.e., "state") of SU. This itself, which we shall hereafter call "Statewise Optimum (SO)" hypothesis, is a reasonably natural extension of the optimization under certainty. Adopting this hypothesis, the optimum consumption vector (C_1^*, C_2^*, C_3^*, …), corresponding to each "state" of SU (i.e., each state of the budget line), will itself be stochastic. Then, there is no asymmetry between C_1^* and C_2^*, C_3^*, …, because all the C^**'s* are stochastic, in contrast to the standard approach adopting AP.

Adopting SO as the main behavioral hypothesis of the consumer, we now intend to reexamine the intertemporal consumption choice under uncertainty. All of the "Statewise Optimum" consumption plans will be symmetrically treated as stochastic. As we shall see, however, SO alone is insufficient to choose the optimum consumption plan (C_1^* in particular) deterministically and uniquely. For this purpose, one must further consider the following two points.

First, the risk preference must be taken into consideration[3]. We shall introduce Neumann-Morgenstern (NM) risk preference function and derive the optimum consumption demand by making use of certainty equivalence. As will be shown later, however, this alone is not yet sufficient to choose a deterministic optimum consumption uniquely, for the introduction of risk preference may only select a group of statewise-optimized certainty equivalent consumption plans with identical preference ordering.

Secondly then, we must introduce an auxiliary hypothesis as a new postulate in order to choose the optimum consumption uniquely. Our new postulate, which substitutes AP, will be introduced in detail in Section 4 but is broadly stated for now as follows: Out of the group of statewise-optimized certainty equivalent consumption plans with identical preference ordering, it is the "mean" of them that is finally chosen as a unique and deterministic consumption plan.

In what follows, Section 2 develops SO, the Statewise Optimization. Section 3 then introduces risk preference and certainty equivalence, and explains what is meant by "a group of optimized certainty equivalent consumption plans with identical preference ordering". Section 4 introduces our new postulate and explains what is meant by the "mean" of this group. Section 5 compares our approach with the standard approach.

Our approach, treating all the C's symmetrically, may be applied not only to the intertemporal consumption choice but also generally to the consumption choice under uncertainty when SU is given as jointly distributed. Further, although intertemporal consumption choice articles have often treated either the price variables (interest rate) or the budget variable as stochastic, this paper treats both variables simultaneously as uncertain. Still further, our approach seems simpler than Selden who postulated "Risk Preference Independence" and "Preference Ordering over Certain-Uncertain Pairs", which themselves seem rather complicated. Our approach may solve the intertemporal

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3 Hall (implicitly) and Selden (explicitly) take the risk preference into consideration. Hall uses a (cardinal) preference ordering which simultaneously works as an NM function.

consumption choice problem without them, and, as will be seen, the extension to more than 2 period model is straightforward.

2: Source Uncertainty SU and Statewise Optimization SO

(Source Uncertainty SU)

In what follows, we shall mainly deal with the two period model unless explicitly stated otherwise. Under PIH, the consumer chooses the optimum intertemporal consumption path which satisfies:

$$\text{Max V}(C_1, C_2) \cdots\cdots\cdots\cdots \quad (1)$$

$$\text{s.t.B} = C_1 + \frac{C_2}{1+r} \cdots\cdots\cdots\cdots \quad (2),$$

where r is the competitive market interest rate from period 1 to 2. B (≡H+N) is the wealth, composed by the human capital H and the financial asset N, both at the beginning of period 1. The human capital is defined as:

$$\text{H} \equiv W_1 + \frac{W_2}{1+r} \cdots\cdots\cdots\cdots \quad (3),$$

where W_1, W_2 (both assumed as stochastic) are the wage income of each period.

If no uncertainty exists, an optimum consumption path (C_1^*, C_2^*), corresponding to a deterministic pair (r, B) and satisfying (1) and (2), will be chosen. Under uncertainty, in contrast, the consumer subjectively thinks that r and B are jointly distributed. The joint distribution (r, B) is our Source Uncertainty (SU)[4].

(Statewise Optimization SO)

Source Uncertainty (r, B) is a joint distribution generating each pair of r and B with specific probability. We write each possible occurrence of (r, B) as (r_J, B_J) and

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[4] Note however that Source Uncertainty is not necessarily “Ultimate Uncertainty”. Although the market price variables (e.g., interest rate) are always the ultimate source of uncertainty under perfect competition, variables such as W (wage income) are not necessarily so. Wage income itself is the product of the market wage rate (i.e., a price variable) and labor supply, and we must consider the latter as optimally chosen by the consumer. Hence, H as defined by (3) is not necessarily “Ultimate Uncertainty”, and B=H+N is not necessarily so either. In this sense, B as a component of SU is an intermediary concept. Treating W (and H, B) as SU is a simplifying assumption of our model which does not treat the leisure-income choice endogenously. We shall refer to this point in Section 4.

the associated probability density as $\pi_J \equiv \pi(r_J, B_J) \geq 0$[5]. Each (r_J, B_J) is a "state" of SU, Source Uncertainty.

Corresponding to each (r_J, B_J), the consumer is able to solve the optimization problem (1)(2) to obtain $(C_1^{*,J}, C_2^{*,J})$, the "Statewise Optimum (SO) Solution". Since (r_J, B_J) occurs with density π_J, the corresponding $(C_1^{*,J}, C_2^{*,J})$ occurs with the same density. Repeating this procedure for each and every state of SU, we may construct a joint distribution (C_1^*, C_2^*), whose J-th state $(C_1^{*,J}, C_2^{*,J})$ occurs with density π_J. Such is what we mean by "Statewise Optimization (SO)".

What is meant by SO? First, it derives the joint distribution (C_1^*, C_2^*) from that of (r, B). Thus derived, (C_1^*, C_2^*) is a set of statewise optimum consumption plans corresponding to each state of SU.

Secondly, SO composes intertemporal consumption plans as stochastic variables. In solving the intertemporal consumption problem, there must generally be a presumption that the consumption plans are stochastic, for otherwise it is logically impossible to carry out stochastic operations such as calculating the expected utility. To derive (C_1^*, C_2^*) as stochastic by applying SO is a logically necessary step. If we do not assume this step, we would have to postulate (rather than logically derive) the consumption choice space[6].

Third, C_1^* and C_2^* are generally stochastic as statewise-optimized "Uncertain-Uncertain" pairs. In contrast, Selden, op.cit., postulates the consumption choice space (C_1, C_2) itself as "Certain-Uncertain Pairs", and consequently, his preference ordering concerns each pair of "Certain C_1" and "Uncertain C_2". We do not employ this AP framework, postulating instead that the consumption space (C_1, C_2) itself is "Certain-Certain Pairs" and that the preference ordering represented by V( ) above concerns "Certain C_1 and Certain C_2" pairs alone. It is the consequence of SO that the pair (C_1^*, C_2^*) becomes "Uncertain-Uncertain Pairs". Consequently, the preference ordering we postulate is simpler than that of Selden's.

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5 For convenience, we assume (r, B) to be continuously distributed.

6 It is in this context that Selden, op.cit. ,which does not employ SO as a behavioral hypothesis under uncertainty, postulates that the consumption choice space (C_1, C_2) itself is "Certain-Uncertain" pairs. We do not employ this postulate. See below.

(Classification of Statewise Optimum Plans)

To simplify notation, we hereafter write $\overrightarrow{C}$ or (C_1, C_2) (instead of $\overrightarrow{C^*}$ or (C_1^*, C_2^*)) to mean the optimum consumption plans derived by SO. $(C_1^J, C_2^J)\equiv\overrightarrow{C^J}$ is the J-th statewise optimum consumption plan corresponding to the J-th state of SU, (r_J, B_J). $\overrightarrow{C^J}$ occurs with density $\pi_J \geq 0$. Our ultimate target is to choose a deterministic vector $\widehat{\overrightarrow{C}}$ uniquely, out of the possible occurrences of the stochastic vector $\overrightarrow{C}$. Let us proceed step-by-step.

For each and every (r_J, B_J), the Statewise Optimization (1)(2) to maximize the ordinal preference function V( ) is carried out to obtain $\overrightarrow{C_J}$, the J-th state of $\overrightarrow{C}$. As mathematical transform, SO is a one-to-one correspondence:

$$\text{SO}: ((\text{r, B}) \mid \pi(\text{r, B})) \leftrightarrow (\overrightarrow{C} \mid \pi(\overrightarrow{C})) \quad \cdots\cdots\cdots\cdots\cdots\cdots \quad (4),$$

where π(r, B) is the density of (r, B), i.e., SU. The J-th state of SU occurs with the density $\pi_J \equiv \pi(r_J, B_J)$, which is numerically equal to $\pi(\overrightarrow{C^J})$, the density of the J-th optimum consumption plan under SO.

In the next step, we make further use of the ordinal preference function V( ) to consider the following mathematical transform:

$$\text{V}: \overrightarrow{C} \rightarrow \text{V}(\overrightarrow{C}) \in R^+$$

This transform is not one-to-one. We then classify $\{\overrightarrow{C}\}$, the set of SO consumption plans, according to the value of $\text{V}(\overrightarrow{C})$, i.e., we now consider the following set Q(V) defined as:

$$\text{Q(V)} \equiv \{\overrightarrow{C} \mid V(\overrightarrow{C}) = V\} \cdot \cdots\cdots\cdots\cdots\cdots\cdots\cdots \quad (5).$$

Notice that Q(V) is an equivalence class whose elements are SO consumption plans with identical preference ordering, V. The density π(V) corresponding to Q(V) is:

$$\pi(\text{V}) = \int_{\overrightarrow{C} \in Q(V)} \pi(\overrightarrow{C})\, d\overrightarrow{C},$$

and we additionally assume π(V) > 0. We therefore obtain, by this step, another stochastic variable V, i.e.,

$$(\text{V} \mid \pi(\text{V})),\ \text{V} \in \text{V}(\overrightarrow{C}).$$

We now proceed to still another step in the next section.

<u>3: Risk Preference and Certainty Equivalence</u>

Up to this stage, we have used the ordinal preference ordering V alone. We now introduce risk preference.

Consider a function Z( ), which transforms V, an ordinal number, into Z(V), a cardinal number. Assume also that $\frac{dZ}{dV}>0$. Z(V) then is an NM function evaluating the risk preference of the consumer.

Using Z, we now define another stochastic variable Z(V) such that:

$$(Z(V) \mid \pi(V)),\ V \in V(\overrightarrow{C}) \quad \cdots\cdots \quad (6)$$

Because Z(V) is cardinal, we may use its expected utility E[Z(V)] to define v^*, the certainty equivalent ordinal utility level corresponding to E[Z(V)]. v^* is defined as satisfying:

$$Z(v^*)=E[Z(V)]\equiv\int_{V(\overrightarrow{C})}Z(V)\pi(V)dV \quad \cdots\cdots \quad (7).$$

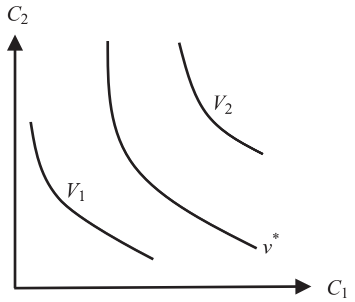

Fig.1

In terms of Fig.1, v^* is the indifference curve over which are distributed SO consumption plans belonging to the same equivalence class Q(v^*). Because v^* is the certainty equivalence of Z(V), it is reasonable to assume that the consumer chooses to behave on the indifference curve v^* rather than stochastically behave on V_1, V_2, *etc*.

The indifference curve v^*, however, contains many SO consumption plans. In order to reach our ultimate objective to single out a deterministic consumption decision on v^*, we still have to proceed to the next step, with which we deal in the next section.

4: Deterministic choice of the consumption plan and a new postulate

To repeat our problem at hand, we wish to find a deterministic consumption plan $\widehat{\overrightarrow{C}}\equiv(\widehat{C_1}, \widehat{C_2})$ on the indifference curve v^* of Fig.1. Once we find $(\widehat{C_1}, \widehat{C_2})$, we simultaneously find $(\widehat{r}, \widehat{B})$, because of (4), i.e., the one-to-one correspondence between SU and the consumption plan derived by SO.

We already know that $(\widehat{C_1}, \widehat{C_2})$ and $(\widehat{r}, \widehat{B})$ should satisfy the following:

$$B=C_1+\frac{C_2}{1+r} \quad \cdots\cdots \quad (8)$$

$$1+r=MRS(C_1, C_2) \quad \cdots\cdots \quad (9)$$

$$v^*=V(C_1, C_2) \quad \cdots\cdots \quad (10),$$

where MRS is the marginal rate of substitution between C_1 and C_2, i.e., $\frac{\partial V/\partial C_1}{\partial V/\partial C_2}$. The equations (8) and (9) are necessary for $(\widehat{C_1}, \widehat{C_2})$ to be optimum with respect to a state (including a state with zero density) of (r, B). The equation (10) requires ($\widehat{C_1}$, $\widehat{C_2}$) to be located on the indifference curve v^*. Taken together, (8),(9)and (10) are to be regarded as simultaneous equations to determine $\widehat{C_1}$, $\widehat{C_2}$, $\widehat{r}$ and $\widehat{B}$. Obviously, however, we need another equation, or a transform T, generally, of the form

$$g(C_1, C_2, r, B) = 0, \text{ or, } \quad T : (C_1, C_2, r, B) \rightarrow R \cdots\cdots\cdots\cdots \quad (11),$$

to determine all the unknowns.

We need to consider what is appropriate and plausible as (11). In order to do so, consider Q(v^*), the set of SO consumption plans which are "v^*-conditionally" distributed over the indifference curve v^*. π(v^*), the density with which v^* occurs, is:

$$\pi(v^*) = \int_{\overrightarrow{C} \in Q(v^*)} \pi(\overrightarrow{C})\, d\overrightarrow{C},$$

which is positive by the assumption π(V)>0, stated earlier. Then we may consider the v^*-conditional stochastic vector $\overrightarrow{C}$ distributed on the indifference curve v^*, to be denoted as ($\overrightarrow{C}\,|\,v^*$). The v^*-conditional density, π($\overrightarrow{C}\,|\,v^*$), is:

$$\pi(\overrightarrow{C}\,|\,v^*) \equiv \frac{\pi(\overrightarrow{C})}{\pi(v^*)} \quad (\text{where, } \overrightarrow{C} \in Q(v^*)).$$

Consider further the pairs (r, B) which generate ($\overrightarrow{C}\,|\,v^*$) as SO. These pairs are distributed on the indirect indifference curve $\widetilde{v}(\ )$ satisfying $v^* = \widetilde{v}(r, B)$. Again because of the one-to-one correspondence (4), they are considered as v^*-conditional joint distribution, to be denoted as (r, B)$|_{v^*}$. The v^*-conditional density π(r, B)$|_{v^*}$ is numerically the same as π($\overrightarrow{C}\,|\,v^*$).

How does the consumer behave on the indifference curve $v^* = V(C_1, C_2)$, or, put equivalently, on the indirect indifference curve $v^* = \widetilde{v}(r, B)$? At this stage, recall that r, the market price variable, is always the Ultimate Uncertainty the competitive consumer faces (as distinguished from Source Uncertainty), but that B is not necessarily so (see footnote 4). Bearing this in mind, we assume that the consumer will regard r that occurs as the v^*-conditional distribution (r, B)$|_{v^*}$ as ultimately relevant to consumption choice on the indifference curve $v^* = V(C_1, C_2)$. Substituting "Asymmetry Postulate (AP)", we now introduce our postulate which is composed of two parts (α) and (β) below. It reads:

α) Consumer on the indirect indifference curve $v^* = \widetilde{v}(r, B)$ seeks a value $\widehat{r}$ included as one of the possible occurrences of (r, B)$|_{v^*}$ and chooses a deterministic pair $(\widehat{C}_1, \widehat{C}_2)$ that satisfies $1+\widehat{r}=MRS(\widehat{C}_1, \widehat{C}_2)$ on the indifference curve $v^*=V(C_1, C_2)$.

β) The consumer determines the value of $\widehat{r}$ by minimizing the expected value of the quadratic loss $E_{v^*}[\mathrm{r}-\widehat{r}]^2$ with respect to $\widehat{r}$, where the expectation E_{v^*} is calculated using the v^*-conditional density $\pi(\mathrm{r}, \mathrm{B})|_{v^*}$[7],which is numerically the same as $\pi(\overrightarrow{C}\,|v^*)$.

According to β), the loss function is minimized when:

$$\widehat{r}=E_{v^*}[\mathrm{r}], \qquad (12),$$ [8]

and we may accordingly determine $\widehat{B}$ as satisfying $v^*=\widetilde{v}(\widehat{r}, B)$.

The equation (12), including a transform from r to $E_{v^*}[\mathrm{r}]$, is indeed a form (11) may possibly take. Combining (12) with (8)(9)(10), we may calculate $\widehat{C}_1$, $\widehat{C}_2$, $\widehat{r}$ and $\widehat{B}$. Further, notice that the pair $(\widehat{C}_1, \widehat{C}_2)$ has been singled out as satisfying $1+\widehat{r}=MRS(\widehat{C}_1, \widehat{C}_2)$ on the indifference curve $v^*=V(C_1, C_2)$, treating both C_1 and C_2 symmetrically as stochastic. Our postulate has made it possible to solve the intertemporal consumption choice problem under uncertainty without using AP.

We may now say what our statement in Section 1 exactly means: i.e., "Out of the group of optimized certainty equivalent consumption plans with identical preference ordering, it is the "mean" of them that is finally chosen as a unique and deterministic consumption plan".

The mean here refers to the mean of MRS on the v^*- conditional indifference curve $v^*=V(C_1, C_2)$. According to (9), each consumption pair on this indifference curve has a one-to-one correspondence with MRS on that curve. Because MRS is chosen optimally equal to each state of $1+\mathrm{r}$ on $v^*=V(C_1, C_2)$, MRS on $v^*=V(C_1, C_2)$ itself is another v^*-

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7 Because the indirect indifference curve $v^*=\widetilde{v}(r, B)$ gives a one-to-one correspondence between r and B, the density of r on this indirect indifference curve is equal to $\pi(\mathrm{r}, \mathrm{B})|_{v^*}$ itself.

8 The loss here referred to is related to the difference between the statewise optimum consumption pairs distributed over the indifference curve $v^*=V(C_1, C_2)$ and the deterministic choice of $(\widehat{C}_1, \widehat{C}_2)$ on the same curve. It is due to the assumed quadratic loss function that $\widehat{r}$ is equal to $E_{v^*}[\mathrm{r}]$. We however offer no strong justification for using the quadratic loss function except that it is by far the most prevalently assumed in the literature. If instead a linear loss function is assumed, $\widehat{r}$ will take on a different value such as the median. What is important from the standpoint of economic analysis though is that the specification of the loss function is irrelevant to the utility level of the agent, because $(\widehat{C}_1, \widehat{C}_2)$ is anyway chosen from the consumption pairs on the indifference curve $v^*=V(C_1, C_2)$.

conditional stochastic variable whose density is numerically equal to $\pi(r, B)|_{v^*}$. If MRS on $v^* = V(C_1, C_2)$ is chosen as satisfying $1 + E_{v^*}[r] = MRS(\widehat{C_1}, \widehat{C_2})$, it simultaneously means that the consumer chooses E_{v^*}[MRS] on the same indifference curve to determine $(\widehat{C_1}, \widehat{C_2})$. The "mean" quoted above is not the mean of (C_1, C_2) on $v^* = V(C_1, C_2)$, but the mean of MRS on the same v^*- conditional indifference curve.

To sum up, solving the intertemporal consumption choice problem generally requires some new behavioral hypotheses (postulates) not used under certainty. One such example is the Asymmetry Postulate (AP) employed by Hall and Selden. As we noted at the outset, however, there is no strong reason to stick to AP. We instead replace AP with our own, and treat each period consumption plans symmetrically as stochastic. It must be added that our approach is readily applicable to more-than-two-period (or, more-than-two-goods) model[9].

5: Comparison with Hall and Selden

Let us compare our approach with AP, in particular Hall[1978] and Selden[1978].

Hall starts his analysis assuming a cardinal utility function $U(C_1, C_2)$. It corresponds to our $Z[V(C_1, C_2)]$, although Z and V are undistinguished by Hall. For Hall, who treats r as non-stochastic, Source Uncertainty SU is simply a given distribution of B.

As we have shown, our SO procedure requires only the statewise budget constraint be satisfied for maximization. For Hall, however, we may interpret that not only the statewise budget constraint but also "C_1 is certain" must be satisfied as the second restriction for maximization. Indeed, Hall's optimum condition is obtained if we

[9] Extension to 3-period model is as follows. Consider the indifference surface $v^* = V(\overrightarrow{C})$, where $\overrightarrow{C} \equiv (C_1, C_2, C_3)$ is the statewise optimum and v^* is the certainty equivalent level of V. Further, let r_i denote the interest rate from period i to period i+1. Denoting by $MRS_{i,j}$ the marginal rate of substitution between C_i and C_J, $(\widehat{C}_1, \widehat{C}_2, \widehat{C}_3)$ is to be chosen as satisfying $E_{v^*}[1+r_1] = MRS_{1,2}$ and $E_{v^*}(1+r_1)(1+r_2) = MRS_{1,3}$ on $v^* = V(\overrightarrow{C})$.

maximize the expected utility under the two constraints[10],[11]. To be sure, the second restriction is a technically possible way to choose C_1 deterministically, but Hall's restriction is stronger than ours, because we impose the statewise budget constraint alone. It must be recognized, then, that Hall's optimum, because it implicitly postulates C_1 asymmetrically as a variable with certainty, is subotimal from the standpoint of our approach.

Further to be noticed is that Hall's approach starts from the cardinal utility U(≡ZV), without distinguishing V from Z. Although it is true that Hall's maximization does take risk preference into account, it is not clear the extent to which the consumption choice is influenced by risk preference. In order to analyze how the risk preference influences the consumption choice explicitly, one must introduce risk preference function separately and make use of the certainty equivalence, as Selden does.

To postulate (C_1, C_2) explicitly and asymmetrically as "Certain-Uncertain Pairs", as Selden does, is another way to start the analysis. Again, this postulate is a possible way to choose C_1 deterministically. Assuming further that risk preference with respect to C_2 is independent of C_1 ("Risk Preference Independence"), Selden makes use of certainty equivalence (of C_2) to choose C_1 deterministically.

As we have remarked at the outset, the assumed asymmetry of C_1 and C_2 (future

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[10] Let U be additively separable, as Hall does. Let the density of each state of B be π(B); δ be the utility discount rate; and C_i (B) (i=1, 2, .., n) be the statewise choice of C_i (i=1, 2, .., n) corresponding to the stochastic B. Given the stochastic budget constraint $B=C_1(B)+\sum_{i=2}^{n}\frac{C_i(B)}{(1+r)^{i-1}}$, Hall's target is to maximize the expected utility

$$EU\equiv\int u[C_1(B)]\pi(B)dB+\sum_{i=2}^{n}\int\frac{u(C_i(B))}{(1+\delta)^{i-1}}\pi(B)dB.$$

According to Hall, the optimum condition is $E_t[u'(C_{t+1})]=\frac{1+\delta}{1+r}u'(C_t)$. For t=1, then,

$$E_1[u'(C_2)]=\frac{1+\delta}{1+r}u'(C_1)\cdots\cdots\cdots\cdots\text{(a)}$$

is required for optimum (Hall [1978], p.974, Theorem).

As we shall discuss in the next footnote, however, the condition (a) is not obtained under the stochastic budget constraint alone.

[11] Consider EU and the stochastic budget constraint in the preceding footnote, and suppose EU is maximized under the stochastic budget constraint alone. Statewise optimization of our approach requires, for each state of B,

$$u'(C_2(B))=\frac{1+\delta}{1+r}u'(C_1(B))\cdots\cdots\cdots\cdots\text{(b)},$$

be necessary, where $C_1(B)$ *and* $C_2(B)$ occur with density π(B). Operating E_1 over (b), we obtain

$$E_1[u'(C_2)]=\frac{1+\delta}{1+r}E_1[u'(C_1)]\cdots\cdots\cdots\cdots\text{(c)}$$

as the implication of our statewise optimality. Particularly noteworthy is that the condition (c) and Hall's optimum condition (a) are generally different (See the preceding footnote and compare RHS).

However, if our statewise optimization is carried out not only under the stochastic budget constraint but also under the second constraint "$C_1(B)=C_1$ for any state of B", then (c) coincides with (a). As suggested in the introduction as well as in the proceeding footnote, it is possible to interpret Hall's optimum condition (a) as obtained with stronger restrictions than ours. See Tanaka-Mutoh[2016].

consumption, generally) is questionable. Consumption choice under uncertainty is a problem to choose the optimum consumption plan when Source Uncertainty, shown as a distribution of (r, B), is given. It is of course required that the current period consumption demand, ($\widehat{C_1}$), be ultimately chosen deterministically, but this purpose does not logically justify nor rationalize to treat theoretically that C_1, as a choice variable for optimum behavior, to be postulated as certain. Selden's approach, however, does employ such a postulate, before any optimization behavior takes place.

Rather, it would be more natural, as we do, to think that the consumption pair is subject to uncertainty because of the Statewise Optimization behavior under Source Uncertainty. For our approach, it is enough if V is assumed as the ordering with respect to "Certain-Certain Pairs", because our maximization is carried out for each state of Source Uncertainty. It is the statewise optimized consumption pairs that are "Uncertain-Uncertain" in our approach.

After obtaining $\overrightarrow{C} \equiv (C_1, C_2)$ by way of Statewsie Optimization, we apply Z over V($\overrightarrow{C}$) to compute v^* as a set of optimum consumption plans reflecting risk preference of the consumer. We then introduce our new postulate (α) and (β) to arrive finally at the deterministic consumption plan under uncertainty. To postulate a preference ordering with respect to "Certain-Uncertain Pairs" is unnecessary. Further, our approach includes the optimization under certainty as a special case, in which (r, B) is considered as a distribution with all the density concentrated at a single point.

Final Remarks

The subject of this paper has been to choose, for a period in historical time, a deterministic consumption plan ($\widehat{C_1}$, $\widehat{C_2}$) (more generally, ($\widehat{C_1}$, $\widehat{C_2}$, $\widehat{C_3}$, …)). Although $\widehat{C_1}$ is the actual demand in period t, $\widehat{C_2}$, $\widehat{C_3}$, etc are the planned consumption as of t, given the information (depending on which the distribution (r, B) is subjectively composed) as of t.

When historical time develops until the next period arrives, new information to revise the distribution of Source Uncertainty (r, B) will have accumulated. Generally, it is not possible to regard $\widehat{C_2}$ as carried out as planned in the former period. It is desirable, then, to analyze the historical development of consumption demand as Stochastic Process when both r and B develop as stochastic process. For this purpose, one must specify how Source Uncertainty historically develops as information accumulates. We leave this, however, as a future research topic[12].

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[12] As well known, Hall[1978] developed, within the framework of the Asymmetric Postulate and treating r as non-stochastic, the famous analysis of the consumption as a stochastic process. Making use of our Symmetric Approach, Tanaka-Mutoh[2016] also attempted to examine the consumption demand as stochastic process, but the interest rate was assumed non-stochastic.

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